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Switching Control for Guaranteeing the Safety of a Tethered Satellite

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There are many problems in a tethered satellite system, such as the danger of the tether being cut by space debris and difficulty in analyzing the tether because of its flexibility. Moreover, because a tether cannot be pushed, control tension is constrained to be nonnegative during deployment and retrieval of the tethered satellite. Therefore, it is difficult to guarantee that the tethered satellite will not collide with the mother ship or that the tether will not entangle itself with the mother ship. A method is proposed to guarantee the safety of a tethered satellite. To operate a tethered satellite safely while avoiding the situations described, a switching control algorithm is constructed for satellite deployment and retrieval. A model in which the tether has neither mass nor flexibility is used to construct a switching control algorithm. Because this model is linear when the dimensionless input is constant, conditions for guaranteeing safety can be derived analytically. Then, the obtained conditions are applied to a simulation with a model in which the tether has mass and flexibility to show the validity of the conditions for guaranteeing safety.

Nomenclature

A	=	cross section of tether, m^2
d	=	dummy input
E	=	Young's modulus of tether, N/m^2
$\boldsymbol{F}(s, t)$	=	elastic force vector in tether
$\boldsymbol{F}'(s, t)$	=	damping force vector in tether
G	=	gravitational constant, $m^3 \cdot kg^{-1} \cdot s^{-2}$
I_n	=	$n \times n$ identity matrix
J	=	performance index
L_f	=	length of tether at target position, m
M	=	mass of Earth, kg
m	=	mass of subsatellite, kg
$\boldsymbol{R}$	=	position vector of tether with respect to Earth
$\boldsymbol{R}_G$	=	position vector of mother ship, $[0, 0, R_G]^T$
R_G	=	radius of circular orbit of mother ship, m
$\boldsymbol{r}$	=	position vector of tether with respect to mother ship
$\boldsymbol{r}_{\text{sat}}$	=	position vector of subsatellite with respect to mother ship, $[x, y, z]^T$
S_f, Q, R	=	weighting matrices of performance index
s	=	coordinate along tether
$T(t)$	=	time-dependent horizon length of performance index, $T_f(1 - e^{-\alpha t})$, $T_f = 500$, $\alpha = 0.5$
u	=	control input (tension of the tether), N
$\boldsymbol{X}$	=	state of subsatellite, $[x, y, z, \dot{x}, \dot{y}, \dot{z}]^T$
$\boldsymbol{X}_0$	=	initial state of subsatellite, $[x_0, y_0, z_0, \dot{x}_0, \dot{y}_0, \dot{z}_0]^T$
$\boldsymbol{X}_f$	=	target state of subsatellite, $[0, 0, L_f, 0, 0, 0]^T$
z	=	coordinate pointing to the center of the Earth from the mother ship
$z_{\min}$	=	minimum value of z to which subsatellite can approach mother ship
η	=	damping coefficient of tether, $N \cdot s/m$
ρ	=	line density of tether, kg/m
Ω	=	angular velocity of mother ship, rad/s
ω	=	angular velocity vector, $[0, -\Omega, 0]^T$
$\times$	=	external product of vectors

Subscript

i	=	ith node on tether

Superscripts

k	=	kth time step of calculation
T	=	transpose
$\hat{}$	=	dimensionless variable

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Introduction

A SPACE tether is a wire or rope that connects a mother ship to a subsatellite in space. A subsatellite connected by a tether is called a tethered satellite, and a system that includes a tethered satellite is called a tethered satellite system. A tethered satellite system has various possible applications in future space missions. For example, it can be utilized for measurements of air density at different altitudes and of magnetic fields and the transport of payloads. However, there are some problems, such as the danger of the tether being cut by space debris and difficulty in analyzing the tether because of its flexibility.

Because a tethered satellite system is an interesting nonlinear system, various studies have been performed concerning the control of a tethered satellite.[1–6] There are also studies[7,8] on the possibility of a tether being cut by space debris. In addition to those studies, an actual mission[9–11] was performed in 1996. However, the tether broke during deployment, and the mission ended in failure. The cause was reported to be that a large portion of the tether was carbonized by the electric discharge generated in the tether. Moreover, it was also reported that no sign of failure was evident until the tether broke.[12]

Consequently, it is difficult to predict when slack or breaking of the tether will occur. If slack or breaking takes place, the tether may wind around the mother ship, or the loose satellite may collide with the mother ship. Therefore, it is important to discuss adequately the guaranteeing of safety. The purpose of our study is to be able to guarantee that a tethered satellite will not collide with the mother ship even if it becomes uncontrollable due to slack or breaking of the tether during deployment or retrieval. In this study, we guarantee that the tethered satellite does not reach the altitude of the mother ship even if the tether becomes slack or breaks. To obtain such a guarantee, we construct a switching algorithm for the control input. The switching control law is derived from a model in which the tether has neither mass nor flexibility because the equation of motion of the model including the mass and flexibility of the tether is too complicated to derive the conditions for guaranteeing safety analytically. In the simulation, we utilize the model including the mass and flexibility of the tether, whereas the control input is calculated from the model disregarding mass and flexibility of the tether. We show that the model, including the mass and flexibility of the tether, demonstrates satisfactory control even when the control input and the conditions for guaranteeing safety are applied based on the model disregarding mass and flexibility of the tether. In this paper, only the retrieval simulation is presented because of the limitation of space and also for the following reason. Retrieval involves a higher possibility of slack of the tether than deployment, and also, if the tether is pulled too much, the tethered satellite may collide with the mother ship.

Mathematical Model

Consider the tethered satellite system shown in Fig. 1. We use a model in which the mass and flexibility of the tether are disregarded to obtain control designs. We call this model the point mass model. We use this model because the equation of motion of a model in which the mass and flexibility of the tether are considered is too complicated to derive the conditions for guaranteeing safety analytically (Appendix A). A merit of using the point mass model is that we can obtain an analytic solution when the dimensionless control input is constant. Then, we derive a method of guaranteeing safety from this analytic solution.

The tethered satellite is deployed from or retrieved by the mother ship toward a target position on the positive z axis with $z = L_f$. The target position can be kept at equilibrium with positive tension in the tether. We assume the following when considering the system.

1) The mother ship is in a circular orbit of radius R_G and angular velocity Ω around the Earth.

2) The masses of the tether and the tethered satellite are much smaller than that of the mother ship.

3) The sizes of the mother ship and the tethered satellite are not considered.

4) Celestial bodies other than the Earth are disregarded.

5) The length of the tether is much shorter than R_G.

We consider the motion of the tethered satellite in a rotating frame fixed on the mother ship with one of its axes, z, always pointing to the center of the Earth. The equation of motion of the tethered satellite can be written as

$$m\frac{\mathrm{d}^2\boldsymbol{r}_{\mathrm{sat}}(t)}{\mathrm{d}t^2} = -\frac{GmM\boldsymbol{R}(t)}{\|\boldsymbol{R}(t)\|^3} + m\boldsymbol{\omega}\times(\boldsymbol{\omega}\times\boldsymbol{R}(t)) - 2m\boldsymbol{\omega}\times\frac{\mathrm{d}\boldsymbol{r}_{\mathrm{sat}}(t)}{\mathrm{d}t} - u(t)\frac{\boldsymbol{r}_{\mathrm{sat}}(t)}{\|\boldsymbol{r}_{\mathrm{sat}}(t)\|} \tag{1}$$

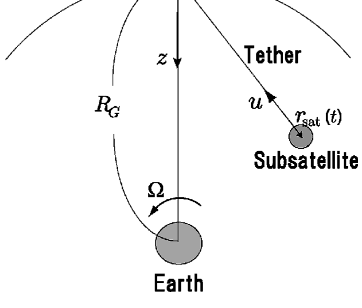

Fig. 1 Tethered satellite system.

Under assumptions 1 and 5, Eq. (1) becomes

$$m\frac{\mathrm{d}^2\boldsymbol{r}_{\mathrm{sat}}(t)}{\mathrm{d}t^2} = -\frac{GmM}{\|\boldsymbol{R}_G\|^3}\left(I_3 - \frac{3\boldsymbol{R}_G\boldsymbol{R}_G^T}{\|\boldsymbol{R}_G\|^2}\right)\boldsymbol{r}_{\mathrm{sat}}(t) + m\boldsymbol{\omega}\times[\boldsymbol{\omega}\times\boldsymbol{r}_{\mathrm{sat}}(t)] - 2m\boldsymbol{\omega}\times\frac{\mathrm{d}\boldsymbol{r}_{\mathrm{sat}}(t)}{\mathrm{d}t} - u(t)\frac{\boldsymbol{r}_{\mathrm{sat}}(t)}{\|\boldsymbol{r}_{\mathrm{sat}}(t)\|} \tag{2}$$

The state equation as is derived from Eq. (2) is as follows:

$$\dot{\boldsymbol{X}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ 2\Omega\dot{z} - \dfrac{x}{m\|\boldsymbol{r}_{\mathrm{sat}}\|}u \\ -\Omega^2 y - \dfrac{y}{m\|\boldsymbol{r}_{\mathrm{sat}}\|}u \\ 3\Omega^2 z - 2\Omega\dot{x} - \dfrac{z}{m\|\boldsymbol{r}_{\mathrm{sat}}\|}u \end{bmatrix} \tag{3}$$

Although the actual control input is the tension of the tether, we make the input dimensionless to simplify the state equation. The relationship between the actual control input u and the dimensionless control input $\hat{u}$ is

$$u = m\|\boldsymbol{r}_{\mathrm{sat}}\|\Omega^2\hat{u} \tag{4}$$

Then, Eq. (3) is rewritten as

$$\dot{\boldsymbol{X}} = \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ 2\Omega\dot{z} - \Omega^2 x\hat{u} \\ -\Omega^2 y - \Omega^2 y\hat{u} \\ 3\Omega^2 z - 2\Omega\dot{x} - \Omega^2 z\hat{u} \end{bmatrix} \tag{5}$$

Although Eq. (5) is not a complicated state equation, it is difficult to deal with this system because it is bilinear. Additionally, because the tether cannot be pushed, the control tension is constrained to be nonnegative, that is, $u, \hat{u} \geq 0$, which makes it difficult to guarantee safety. However, if $\hat{u}$ is constant, Eq. (5) becomes a linear system and we can obtain the analytic solution, which can be utilized to guarantee safety, as is described in the next section. When $\hat{u} = c$ (constant), the state equation becomes linear as follows:

$$\dot{\boldsymbol{X}} = A\boldsymbol{X} \tag{6}$$

where

$$A = \begin{bmatrix} \boldsymbol{0} & I_3 \\ -\Omega^2\begin{pmatrix} c & 0 & 0 \\ 0 & c+1 & 0 \\ 0 & 0 & c-3 \end{pmatrix} & 2\Omega\begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ -1 & 0 & 0 \end{pmatrix} \end{bmatrix} \tag{7}$$

Let $\boldsymbol{X}_0$ be the state vector at the moment $(t = 0)$ of switching the dimensionless control input to $\hat{u} = c$. Then the analytic solution is given by $\boldsymbol{X}(t) = e^{At}\boldsymbol{X}_0$, which can be utilized to estimate the reachable region of the tethered satellite.

It is clear from state equation (5) that although the influence of tension also appears in motion out of the orbital plane, y, the motion in the y direction is independent of that in the x–z plane. Moreover, y motion does not affect the altitude of the tethered satellite. Therefore, we derive the conditions of the state for guaranteeing safety using the motion in the x–z plane and we disregard motion out of the orbital plane hereafter.

Method of Guaranteeing Safety

The worst case in the control of the tethered satellite is that the satellite becomes uncontrollable due to the tension of the tether falling to zero as a result of slack or cutting. When the tether slackens, we can regain control of the satellite by pulling the tether. However, if the tether is pulled beyond necessity, the satellite may collide with the mother ship or the tether may wind around the mother ship. Moreover, if the tether breaks, control of the satellite will become impossible.

To avoid this situation, we first apply constraints to the state of the satellite. That is, we set two constraints to the control law: $u \geq 0$ and $z(t) \geq z_{\min}$ with $L_f \geq z_{\min} \geq 0$. If $z(t) \geq z_{\min}$ is always satisfied, the satellite never collides with the mother ship at $z = 0$ during deployment/retrieval toward $z = L_f$. However, there is no guarantee that these two constraints of the control input and the state can always be satisfied simultaneously. In particular, when the tension of the tether falls to zero and the tether slackens, it becomes impossible to constrain the state because the satellite becomes uncontrollable. Then, in this paper, we stipulate that there is no danger of the satellite colliding with the mother ship even when the satellite becomes uncontrollable. That is, we guarantee safety by guaranteeing $z(t) \geq z_{\min}(z_{\min} \geq 0)$ for all $t \geq 0$ even when the tension of the tether falls to zero. Hence, our main concern is the value of $z(t)$.

Here, we investigate analytic solutions to be used in guaranteeing safety. First, we calculate the control input $\hat{u}_f$ required to maintain equilibrium in the target state. For the target state $\boldsymbol{X}_f = [0, 0, L_f, 0, 0, 0]^T$, the equilibrium condition $\dot{\boldsymbol{X}} = 0$ for Eq. (5) leads to $\hat{u}_f = 3$. Next, we consider motion with a constant dimensionless input $\hat{u} = c$. If $c \geq \hat{u}_f$, the analytic solution for $z(t)$ after switching at $t = 0$ is given by

$$z(t) = \frac{(p+\Omega)\dot{z}_0 - 4\Omega^2 c x_0}{2p\xi_1}\sin\xi_1 t + \frac{(p-7\Omega)z_0 + 4\dot{x}_0}{2p}\cos\xi_1 t + \frac{(p-\Omega)\dot{z}_0 + 4\Omega^2 c x_0}{2p\xi_2}\sin\xi_2 t + \frac{(p+7\Omega)z_0 - 4\dot{x}_0}{2p}\cos\xi_2 t \tag{8}$$

where

$$p = \Omega\sqrt{16c+1}, \qquad \xi_1 = \Omega\sqrt{\tfrac{1}{2}[(p/\Omega)+1]+c}$$

$$\xi_2 = \Omega\sqrt{\tfrac{1}{2}[(p/\Omega)-1]+c} \tag{9}$$

It is clear from Eq. (8) that the satellite vibrates around $z = 0$. That is, the satellite periodically reaches the altitude of the mother ship when $c \geq \hat{u}_f$. Therefore, we only consider the case of $c < \hat{u}_f$ to guarantee safety. If $c < \hat{u}_f$, the analytic solution $z(t)$ is given by

$$z(t) = F\sin(\zeta_1 t + \phi) + [(h+k)/p]e^{\zeta_2 t} + [(h-k)/p]e^{-\zeta_2 t} \tag{10}$$

where

$$p = \Omega\sqrt{16c+1}, \qquad \zeta_1 = \Omega\sqrt{\frac{1}{2}\left[\left(\frac{p}{\Omega}\right)+1\right]+c}$$

$$\zeta_2 = \Omega\sqrt{\frac{1}{2}\left[\left(\frac{p}{\Omega}\right)-1\right]-c}, \qquad h = \frac{p+7\Omega}{4}z_0 - \dot{x}_0$$

$$k = \frac{\Omega^2 c}{\zeta_2}x_0 + \frac{p-\Omega}{4\zeta_2}\dot{z}_0, \qquad \phi = \tan^{-1}\frac{(p-7\Omega)\zeta_1 z_0 + 4\zeta_1\dot{x}_0}{(p+\Omega)\dot{z}_0 - 4\Omega^2 c x_0}$$

$$F = \frac{1}{2p}\sqrt{[4\dot{x}_0 + (p-7\Omega)z_0]^2 + \left[\frac{(p+\Omega)\dot{z}_0 - 4\Omega^2 c x_0}{\zeta_1}\right]^2} \tag{11}$$

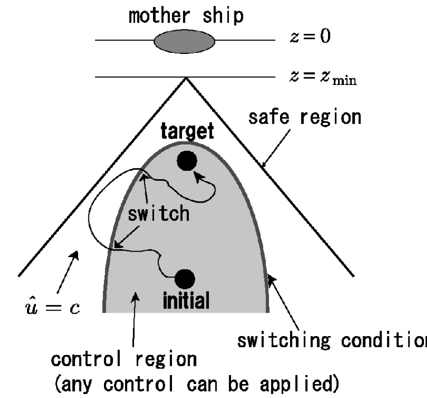

Fig. 2 Method of guaranteeing safety.

Equation (10) has a diverging term due to $\zeta_2 > 0$. Therefore, $z(t)$ can be maintained above a certain value and we can guarantee safety on the basis of Eq. (10).

The method of guaranteeing safety by applying the analytic solution is shown in Fig. 2. In this work, we aim at guaranteeing $z(t) \geq z_{\min}$ ($z_{\min} \geq 0$) for all $t \geq 0$, where $z = 0$ is the altitude of the mother ship. First, we identify the region of the state within which the satellite does not reach $z = z_{\min}$ whenever the tether slackens or breaks, that is, $\hat{u} = 0$, which corresponds to $c = 0$. We call such a region the safe region. If we can constrain the state of the satellite to within the safe region, we can guarantee safety. However, there are some obstacles to guaranteeing safety because the system is nonlinear, and moreover, there is a constraint that the tether cannot be pushed. Then, we utilize the analytic solution obtained from Eq. (6) and $c < \hat{u}_f$.

For the satellite to remain in the safe region, we derive the switching control law using analytic solution (10). The region used by the switching control law is called the control region. The control region is that in which the satellite does not leave the safe region as long as $\hat{u} = c$. The boundary of the control region gives the switching conditions.

While the satellite is in the control region, we can use any control law. When the satellite reaches the boundary of the control region, the control input is switched to $\hat{u} = c$. Because the satellite's motion after switching is based on analytic solution (10), it can be guaranteed that the satellite does not leave the safe region. When the satellite returns to the control region again, the control law returns again to the earlier control law. Then the satellite is guaranteed to always remain in the safe region. Therefore, the satellite does not reach the altitude $z = z_{\min}$ even when the tether slackens or is cut.

In this regard, we pay attention to the following. After the satellite leaves the control region, it is difficult to guarantee theoretically that the satellite will return to the control region, although the satellite returns to the control region in the simulated results. However, the main purpose of our study is to guarantee that the satellite does not collide with the mother ship, that is, $z(t) \geq z_{\min}$, $\forall t \geq 0$. We propose that by switching the control algorithm, $z(t) \geq z_{\min}$ can always be guaranteed.

Safe Region

If the tether slackens or breaks, the satellite begins free motion. Hence, the safe region is equivalent to the region of the state in which the satellite does not reach $z = z_{\min}$ under free motion. The analytic solution of free motion, $c = 0$, in the x–z plane from the initial state $[x_0, z_0, \dot{x}_0, \dot{z}_0]^T$ is given as the solution of Hill's equation (see Ref. 13),

$$x(t) = -(2\dot{z}_0/\Omega)\cos\Omega t + \dot{x}_0 t + x_0 + (2\dot{z}_0/\Omega)$$

$$z(t) = (\dot{z}_0/\Omega)\sin\Omega t + z_0 \tag{12}$$

From Eq. (12), the necessary and sufficient condition for $z(t) \geq z_{\min}$ for all $t \geq 0$ can be found as follows:

$$|\dot{z}_0| \leq \Omega(z_0 - z_{\min}) \tag{13}$$

Consequently, the satellite does not reach $z = z_{\min}$ whenever the tether slackens or breaks if the state of the satellite satisfies the following inequality:

$$C(\boldsymbol{X}) = |\dot{z}(t)| - \Omega(z(t) - z_{\min}) \le 0, \qquad \forall t \ge 0 \tag{14}$$

Inequality (14) corresponds to the safe region in Fig. 2. Because Eq. (14) must always be satisfied, it is a natural idea to impose Eq. (14) as a constraint. However, there is another constraint that $\hat{u}$ must be nonnegative. Therefore, we examine whether Eq. (14) can always be satisfied with a nonnegative control input. That is, we check whether $\hat{u} \ge 0$ exists while satisfying $\dot{C}(\boldsymbol{X}) \le 0$ when the state reaches the boundary of the safe region, $C(\boldsymbol{X}) = 0$. Here, $\dot{C}(\boldsymbol{X})$ is calculated as

$$\dot{C}(\boldsymbol{X}) = \frac{\partial C(\boldsymbol{X})}{\partial \boldsymbol{X}} \dot{\boldsymbol{X}} = [0,\ 0, -1, 0, 0, \pm 1]\dot{\boldsymbol{X}} \tag{15}$$

where the sign of ± 1 depends on the sign of $\dot{z}$ and $\dot{\boldsymbol{X}}$ is given by Eq. (5). As a result, the value of the control input that must be adopted at the boundary such that $\dot{C}(\boldsymbol{X}) \le 0$ holds is obtained as

$$\hat{u} \ge 2 + (z_{\min}/z) - (2\dot{x}/\Omega z) \qquad (\text{if } \dot{z} \ge 0) \tag{16}$$

$$\hat{u} \le 2 + (z_{\min}/z) - (2\dot{x}/\Omega z) \qquad (\text{if } \dot{z} < 0) \tag{17}$$

A nonnegative control input $\hat{u} \ge 0$ that satisfies Eq. (16) always exists. However, for $\hat{u} \ge 0$ that satisfies Eq. (17) to exist, the state of the satellite must satisfy the following inequality:

$$\dot{x} \le \Omega z + (\Omega z_{\min}/2) \tag{18}$$

To always satisfy Eq. (14), the state must satisfy Eq. (18) when $\dot{z} < 0$ and $C(\boldsymbol{X}) = 0$. Similarly, to satisfy Eq. (18) with nonnegative control input, the state must satisfy the newly generated constraints. Although we do not show details because of the limitation of space, new constraints arise one after another to satisfy the preceding constraints, and as a result, it cannot be guaranteed that Eq. (14) is always satisfied with nonnegative control input. Hence, it is necessary to utilize the control region to guarantee safety.

Control Region

Next, we derive the conditions of the initial state $\boldsymbol{X}_0$ to continue to satisfy Eq. (14) under $\hat{u} = c$. For this, the analytic solution (10) is applicable. Equation (10) is differentiated with respect to time and substituted into Eq. (14) to obtain the following conditions.

If $\dot{z}(t) \ge 0$,

$$-\zeta_1 F \cos(\zeta_1 t + \phi) + \Omega F \sin(\zeta_1 t + \phi) + [(h+k)/p](\Omega - \zeta_2)e^{\zeta_2 t} + [(h-k)/p](\Omega + \zeta_2)e^{-\zeta_2 t} - \Omega z_{\min} \ge 0 \tag{19}$$

If $\dot{z}(t) < 0$,

$$\zeta_1 F \cos(\zeta_1 t + \phi) + \Omega F \sin(\zeta_1 t + \phi) + [(h+k)/p](\Omega + \zeta_2)e^{\zeta_2 t} + [(h-k)/p](\Omega - \zeta_2)e^{-\zeta_2 t} - \Omega z_{\min} \ge 0 \tag{20}$$

Note that parameters in Eqs. (19) and (20) depend on c and $\boldsymbol{X}_0$, as shown in Eq. (11). The sufficient condition of $\boldsymbol{X}_0$ such that Eqs. (19) and (20) are satisfied at any time can be derived as (see Appendix B for derivation)

$$h + k > 0 \tag{21}$$

If $\dot{z} \ge 0$, $k\Omega - h\zeta_2 < 0$, or $\dot{z} < 0$, $k\Omega + h\zeta_2 < 0$,

$$\left[2\sqrt{(h^2 - k^2)(\Omega^2 - \zeta_2^2)}\Big/p\right] - F\sqrt{\Omega^2 + \zeta_1^2} \ge \Omega z_{\min} \tag{22}$$

If $\dot{z} \ge 0$, $k\Omega - h\zeta_2 \ge 0$, or $\dot{z} < 0$, $k\Omega + h\zeta_2 \ge 0$,

$$[2(h\Omega + k\zeta_2)/p] - F\sqrt{\Omega^2 + \zeta_1^2} \ge \Omega z_{\min} \tag{23}$$

Inequalities (21–23) define the control region in Fig. 2, and the boundary of the control region gives switching conditions for guaranteeing safety.

In summary, we propose to give switching conditions for the control input. When the state of the satellite is within the control region defined by Eqs. (21–23), the control input is determined according to the applied control law, and when the state reaches the boundary of the control region, the control input is maintained at $\hat{u} = c$. Then the tethered satellite remains in the safe region as long as the tether is undamaged. Consequently, $z(t) \ge z_{\min}$, $\forall t \ge 0$, can be guaranteed whenever the tether slackens or breaks.

Note that the present analysis is valid regardless of y motion because the motion in the x–z plane is decoupled from y motion, as mentioned in the preceding section. Therefore, the distance between the tethered satellite and the mother ship is always greater than or equal to $z_{\min}$ and, consequently, the tethered satellite never collides with the mother ship even when y motion oscillates or diverges. Moreover, the safe region defined by Eq. (14) and the control region defined by Eqs. (21–23) are independent of the mass of the satellite and depends only on $z_{\min}$, Ω, and c. Therefore, the proposed switching conditions are not affected by the mass of the satellite, whereas the actual input is affected by the mass of the satellite, as shown in Eq. (4).

Simulation

Receding Horizon Control

The most important feature of the proposed method of guaranteeing safety is that this method does not depend on the control law to be applied. In this paper, we apply nonlinear receding horizon control (model predictive control) as the control law effective for the nonlinear system and employ a fast algorithm.[14,15] Receding horizon control is a kind of optimal control that minimizes the performance index with a moving horizon and can be executed with only a moderate amount of computation because the moving horizon is shorter than the total time required in control.

Because the control input has the inequality constraint $\hat{u} \ge 0$, the inequality constraint is transformed into an equality constraint $\hat{u} - \hat{d}^2 = 0$ by introducing a dummy input $\hat{d}$ that can be handled by the algorithm. In the calculation of the control law, we use the dimensionless states. The performance index is given as

$$J = \frac{1}{2}\left[\hat{\boldsymbol{X}}(t+T) - \hat{\boldsymbol{X}}_f\right]^T S_f \left[\hat{\boldsymbol{X}}(t+T) - \hat{\boldsymbol{X}}_f\right] + \frac{1}{2}\int_t^{t+T} \left[(\hat{\boldsymbol{X}} - \hat{\boldsymbol{X}}_f)^T Q (\hat{\boldsymbol{X}} - \hat{\boldsymbol{X}}_f) + (\hat{\boldsymbol{v}} - \hat{\boldsymbol{v}}_f)^T R (\hat{\boldsymbol{v}} - \hat{\boldsymbol{v}}_f)\right] \mathrm{d}t' \tag{24}$$

where $\hat{\boldsymbol{X}} = [x/L_f, y/L_f, z/L_f, \dot{x}/L_f\Omega, \dot{y}/L_f\Omega, \dot{z}/L_f\Omega]^T$, $\hat{\boldsymbol{X}}_f = [0, 0, 1, 0, 0, 0]^T$, $\hat{\boldsymbol{v}} = [\hat{u}, \hat{d}]^T$, and $\hat{\boldsymbol{v}}_f = [3, \sqrt{3}]^T$. The horizon length T may depend on time in general. At each time t, the finite horizon optimal-control problem is solved with the initial state given by $x(t)$, and only the initial value of optimal control over the horizon is given as the actual control input, which results in a kind of state feedback control.

Compensation of Mass of Tether

As well as the switching conditions for guaranteeing safety, the control input is also determined by the point mass model disregarding the mass and flexibility of the tether and is applied to the model including mass and flexibility, which gives rise to the following problem. If the control input determined by the point mass model is applied directly to the model including mass and flexibility of the tether, the equilibrium position of the tethered satellite shifts due to the mass of the tether, and satisfactory control of the satellite cannot be achieved. Therefore, to compensate for the mass of the tether, the following operation is performed for the control input.

At the equilibrium position $\boldsymbol{X}_f = [0, 0, L_f, 0, 0, 0]^T$, the external forces are the gravitational force and the centrifugal force. Hence, the total external force over the tether at equilibrium is expressed as

$$\int_0^{L_f}\left(\frac{2GM\rho}{R_G^3}z+\rho\Omega^2 z\right)\mathrm{d}z=\frac{3\Omega^2}{2}\rho L_f^2 \tag{25}$$

The external force applied to the tethered satellite is given by

$$\left(2GMm\big/R_G^3\right)L_f+\rho\Omega^2 mL_f=3\Omega^2 mL_f \tag{26}$$

Then, the total external force on both the tether and the tethered satellite is expressed as

$$3\Omega^2\left(m+\tfrac{1}{2}\rho L_f\right)L_f \tag{27}$$

By comparing Eq. (27) with Eq. (26), we find that if one-half of the mass of the tether is added to the mass of the tethered satellite, the mass of the tether can be compensated for in the sense that the equilibrium position of the tethered satellite remains the same as that of the point mass model. Therefore, we modify the relation of $\hat{u}$ and u in Eq. (4) as

$$u=[m+(\rho\|\boldsymbol{r}_{\mathrm{sat}}\|/2)]\,\|\boldsymbol{r}_{\mathrm{sat}}\|\Omega^2\hat{u} \tag{28}$$

Satisfactory results are obtained with this simple compensation, as shown in simulations presented next.

Retrieval Simulation

Simulations are performed using the model including mass and flexibility of the tether and applying the switching conditions for guaranteeing safety obtained from the point mass model; we verify the validity of the model. See Appendix C for the method of numerical computation of the simulation. Simulation results are shown in Figs. 3–10. The parameters employed in this study are shown in Table 1; they were taken from Cosmo and Lorenzini[16] and Koakutsu.[17] The objective of this simulation is to keep the satellite under $z_{\min}=200$ m during retrieval toward $z=300$ m. Other parameters are $\boldsymbol{X}_0=[250,-100,700,0,0,0]^T$, $\boldsymbol{X}_f=[0,0,300,0,0,0]^T$, $S_f=\mathrm{diag}[10,10,50,10,10,10]$, $Q=\mathrm{diag}[1,2,3,1,1,1]$, and $R=0.1I_2$. The constant dimensionless input is $c=2.8$, and each simulation time is 6 h. In Figs. 3–10, initial represents the initial position and target represents the target position. The dashed lines in Figs. 3–10 show trajectories where the control input is switched to $\hat{u}=c$.

Figures 3 and 4 show the solution trajectory and time history without the switching conditions, respectively. Figure 3 indicates

Table 1 Parameters

Parameter	Value	Unit
A	1.32×10^{-6}	m^2
E	9.80×10^{9}	N/m^2
m	10.0	kg
R_G	6.60×10^{6}	m
η	1.00×10^{-3}	N · s/m
ρ	1.80×10^{-3}	kg/m
Ω	1.18×10^{-3}	rad/s

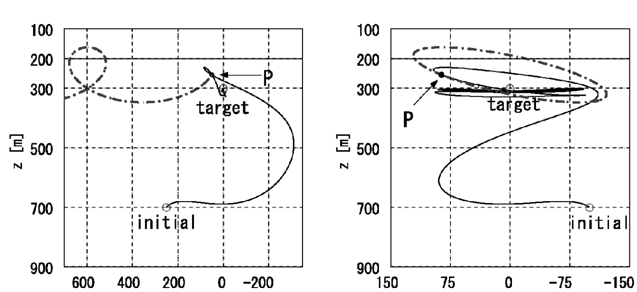

Fig. 3 Model including mass and flexibility, *x*–*z* and *y*–*z* trajectories for nonswitching case.

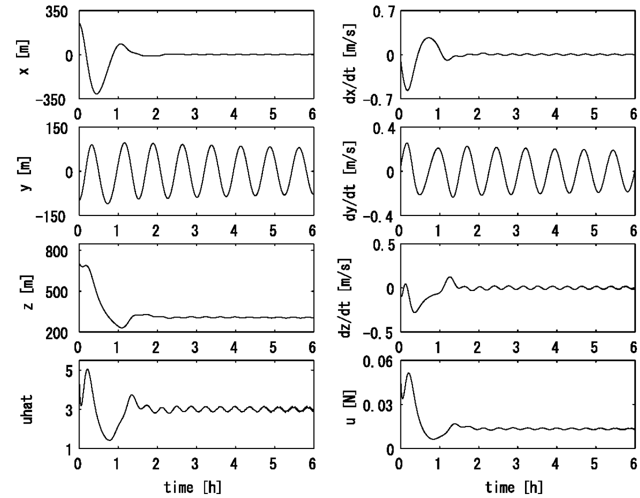

Fig. 4 Model including mass and flexibility, time history for nonswitching case.

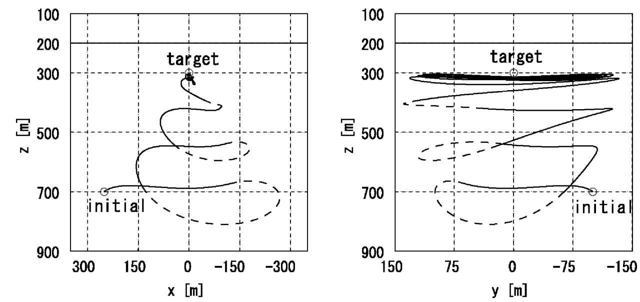

Fig. 5 Model including mass and flexibility, *x*–*z* and *y*–*z* trajectories for switching case.

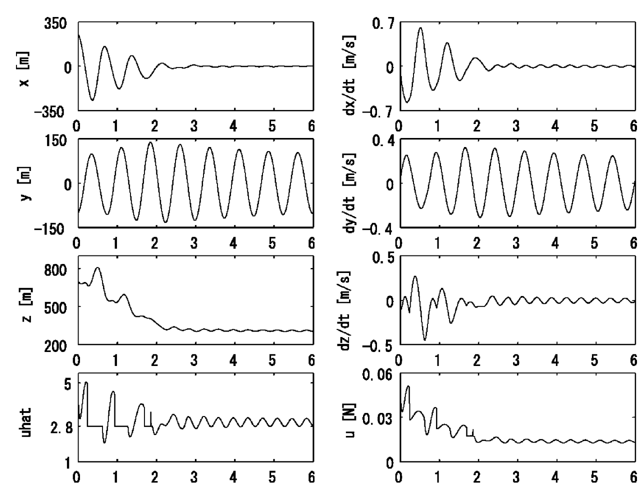

Fig. 6 Model including mass and flexibility, time history for switching case.

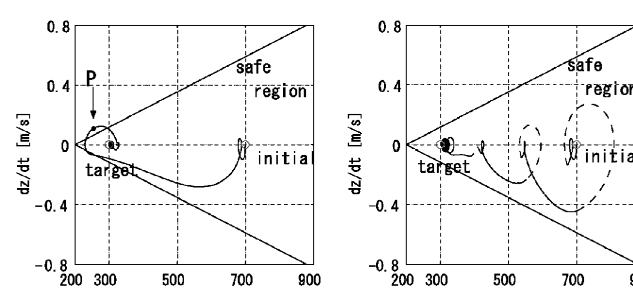

Fig. 7 Model including mass and flexibility, *z*–$\dot{z}$ trajectories for non-switching and switching cases.

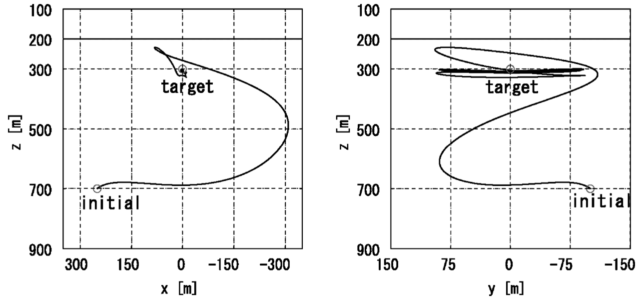

Fig. 8 Point mass model, *x–z* and *y–z* trajectories for nonswitching case.

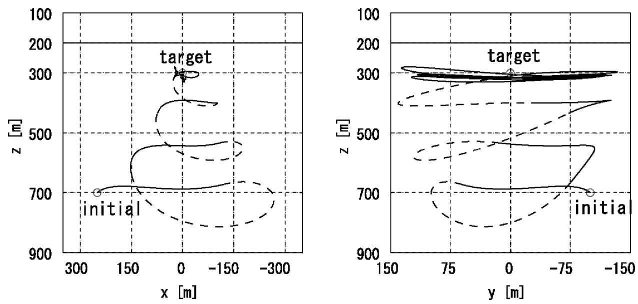

Fig. 9 Point mass model, *x–z* and *y–z* trajectories for switching case.

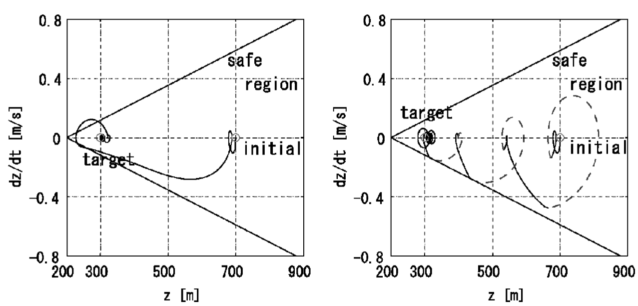

Fig. 10 Point mass model, *z–ż* trajectories for nonswitching and switching cases.

that, if we assume that the tether does not break, retrieval is safe even without switching. However, if the tether breaks at point P shown in Fig. 3, the satellite will enter the region of $z < z_{\min}$, as shown by the ·– line. Then, for the satellite not to enter the region of $z < z_{\min}$, we apply the switching control law using the control region to the dimensionless control input. Figures 5 and 6 show the solution trajectory and time history with the switching conditions, respectively.

We explain the difference in detail between the cases of switching and nonswitching utilizing Fig. 7, which shows the solution trajectories in the z–$\dot{z}$ plane. The region between the two straight lines in Fig. 7 represents the safe region, $C(\boldsymbol{X}) \leq 0$, defined by Eq. (14). That is, constraining the state of the satellite to fall between the two straight lines guarantees safety. Figure 7a shows the result without the switching conditions for the control input and Fig. 7b shows the result with switching. If the tether slackens or breaks in the region outside the two straight lines, the satellite will enter the region of $z < z_{\min}$. In Fig. 7b, because the state always remains in the region between the two straight lines, the satellite does not enter the region of $z < z_{\min}$ whenever the tether slackens or breaks.

In both cases of nonswitching and switching, the convergence of the motion out of the orbital plane y is much slower than that in the orbital plane, x–z plane, because we prioritize the control response in the x–z plane to demonstrate the switching control law. As already mentioned, the proposed switching conditions guarantee safety even when y motion oscillates or diverges. The response out of the orbital plane can be improved by increasing the weights for y and $\dot{y}$ in the performance index at the expense of the response in the orbital plane.

The simulated results using the point mass model are shown in Figs. 8–10 to evaluate the influence of the mass and flexibility of the tether. Figure 8 shows the solution trajectory without the switching conditions, Fig. 9 shows the solution trajectory with the switching conditions, and Fig. 10 shows the solution trajectories in the z–$\dot{z}$ plane. The trajectories of the satellite almost conform to those of the model including mass and flexibility of the tether. Moreover, from the results for the nonswitching case, the maximum lateral displacement of the tether is 1.72 m for the tether length of 493.91 m, which corresponds to 0.35%. From the results for the switching case, the maximum lateral displacement of the tether is 2.69 m for the tether length of 630.89 m, which corresponds to 0.43%. Hence, the influence of flexibility is very small and the difference between the two models is slight. Therefore, the influence of mass and flexibility is so small that the proposed switching control law constructed for the point mass model is sufficiently effective for the realistic model that includes the mass and flexibility of the tether.

Conclusions

To avoid the situation of a tethered satellite colliding with the mother ship, we constructed a switching algorithm for control input. That is, the switching conditions were imposed on the control input by applying the analytic solution for a constant dimensionless input. The switching conditions were calculated using the point mass model, and simulations were performed for the model including mass and flexibility of the tether. As shown by the simulated results, the influence of flexibility was small, and satisfactory performance was obtained. By virtue of the switching conditions, the satellite does not collide with the mother ship even if the tether slackens or breaks and the satellite becomes uncontrollable. That is, the region of motion of the satellite is guaranteed to be safe.

Appendix A: Model Including Mass and Flexibility of the Tether

We summarize the partial differential equation of the model including mass and flexibility of the tether. We assume that the tether has elastic force and damping force, $\boldsymbol{F}(s,t)$ and $\boldsymbol{F}'(s,t)$, respectively, that are expressed as follows:

$$\begin{aligned}
\boldsymbol{F}(s,t) &= EA\frac{\|\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)\|-\Delta s}{\Delta s} \\
&\quad\cdot\frac{\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)}{\|\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)\|} \\
&= EA\left(\frac{\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)}{\Delta s}-\frac{\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)}{\|\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)\|}\right) \\
&\to EA\frac{\partial\boldsymbol{r}(s,t)}{\partial s}\left(1-\left\|\frac{\partial\boldsymbol{r}(s,t)}{\partial s}\right\|^{-1}\right) \qquad (\Delta s\to 0)
\end{aligned} \tag{A1}$$

$$\begin{aligned}
\boldsymbol{F}'(s,t) &= \eta A\frac{\partial}{\partial t}\frac{\|\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)\|-\Delta s}{\Delta s} \\
&\quad\cdot\frac{\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)}{\|\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)\|} \\
&= \eta A\frac{\partial}{\partial t}\left(\left\|\frac{\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)}{\Delta s}\right\|-1\right) \\
&\quad\cdot\frac{\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)}{\|\boldsymbol{r}(s+\Delta s,t)-\boldsymbol{r}(s,t)\|}\to\eta A\frac{\partial}{\partial t}\left(\left\|\frac{\partial\boldsymbol{r}(s,t)}{s}\right\|\right) \\
&\quad\cdot\left(\frac{\partial\boldsymbol{r}(s,t)}{\partial s}\right)\left\|\frac{\partial\boldsymbol{r}(s,t)}{\partial s}\right\|^{-1} \qquad (\Delta s\to 0)
\end{aligned} \tag{A2}$$

Hence, the equation of motion of an infinitesimal portion Δs of the tether is derived as

$$\rho\Delta s\frac{\partial^2 \boldsymbol{R}(s,t)}{\partial t^2} = -\frac{GM\rho\Delta s\boldsymbol{R}(s,t)}{\|\boldsymbol{R}(s,t)\|^3} + \rho\Delta s\boldsymbol{\omega}\times[\boldsymbol{\omega}\times\boldsymbol{R}(s,t)] - 2\rho\Delta s\boldsymbol{\omega}\times\frac{\partial\boldsymbol{R}(s,t)}{\partial t} + \boldsymbol{F}(s+\Delta s,t) - \boldsymbol{F}(s,t) + \boldsymbol{F}'(s+\Delta s,t) - \boldsymbol{F}'(s,t) \tag{A3}$$

Because the order of $(\|\boldsymbol{r}\|^2/\|\boldsymbol{R}_G\|^2)$ can be neglected, the term of gravitational force can be transformed as follows[18]:

$$\frac{\boldsymbol{R}(t)}{\|\boldsymbol{R}(t)\|^3} = \frac{\boldsymbol{R}_G + \boldsymbol{r}_{\text{sat}}(t)}{\|\boldsymbol{R}_G + \boldsymbol{r}_{\text{sat}}(t)\|^3} \simeq \frac{1}{\|\boldsymbol{R}_G\|^3}\left\{\boldsymbol{R}_G + \boldsymbol{r}_{\text{sat}}(t) - \frac{3\left[\boldsymbol{R}_G^T\boldsymbol{r}_{\text{sat}}(t)\right]\boldsymbol{R}_G}{\|\boldsymbol{R}_G\|^2}\right\} = \frac{\boldsymbol{R}_G}{\|\boldsymbol{R}_G\|^3} + \frac{1}{\|\boldsymbol{R}_G\|^3}\left(I_3 - \frac{3\boldsymbol{R}_G^T\boldsymbol{R}_G}{\|\boldsymbol{R}_G\|^2}\right)\boldsymbol{r}_{\text{sat}}(t) \tag{A4}$$

When both sides of the equation of motion are divided with Δs, and taking the limit of $\Delta s \to 0$, the partial differential equation of the tether is obtained as

$$\rho\frac{\partial^2\boldsymbol{r}(s,t)}{\partial t^2} = -\frac{GM\rho}{\|\boldsymbol{R}_G\|^3}\left(I_3 - \frac{3\boldsymbol{R}_G\boldsymbol{R}_G^T}{\|\boldsymbol{R}_G\|^2}\right)\boldsymbol{r}(s,t) + \boldsymbol{\omega}\times[\boldsymbol{\omega}\times\boldsymbol{r}(s,t)] - 2\rho\boldsymbol{\omega}\times\frac{\partial\boldsymbol{r}(s,t)}{\partial t} + \frac{\partial\boldsymbol{F}(s,t)}{\partial s} + \frac{\partial\boldsymbol{F}'(s,t)}{\partial s} \tag{A5}$$

where

$$\frac{\partial\boldsymbol{F}(s,t)}{\partial s} = EA\left(I_3 - \frac{I_3}{\|\boldsymbol{A}\|} + \frac{\boldsymbol{A}\boldsymbol{A}^T}{\|\boldsymbol{A}\|^3}\right)\times\boldsymbol{B}$$

$$\frac{\partial\boldsymbol{F}'(s,t)}{\partial s} = \eta A\left[\left(\frac{\boldsymbol{B}\boldsymbol{A}^T}{\|\boldsymbol{A}\|^2} - \frac{\boldsymbol{A}\boldsymbol{A}^T\boldsymbol{B}\boldsymbol{A}^T}{\|\boldsymbol{A}\|^4} + \frac{\boldsymbol{A}\boldsymbol{B}^T}{\|\boldsymbol{A}\|^2} - \frac{\boldsymbol{A}\boldsymbol{A}^T\boldsymbol{A}^T\boldsymbol{B}}{\|\boldsymbol{A}\|^4}\right)C + \frac{\boldsymbol{A}\boldsymbol{A}^T}{\|\boldsymbol{A}\|^2}\boldsymbol{D}\right]$$

$$\boldsymbol{A} = \frac{\partial\boldsymbol{r}(s,t)}{\partial s},\qquad \boldsymbol{B} = \frac{\partial^2\boldsymbol{r}(s,t)}{\partial s^2},\qquad \boldsymbol{C} = \frac{\partial^2\boldsymbol{r}(s,t)}{\partial s\partial t}$$

$$\boldsymbol{D} = \frac{\partial^3\boldsymbol{r}(s,t)}{\partial s^2\partial t} \tag{A6}$$

The boundary condition at $s=\ell$ is given by the equation of motion for the tethered satellite as follows:

$$m\frac{\partial^2\boldsymbol{r}(\ell,t)}{\partial t^2} = -\frac{GmM}{\|\boldsymbol{R}_G\|^3}\left(I_3 - \frac{3\boldsymbol{R}_G\boldsymbol{R}_G^T}{\|\boldsymbol{R}_G\|^2}\right)\boldsymbol{r}(\ell,t) + m\boldsymbol{\omega}\times[\boldsymbol{\omega}\times\boldsymbol{r}(\ell,t)] - 2m\boldsymbol{\omega}\times\frac{\partial\boldsymbol{r}(\ell,t)}{\partial t} - EA\left(\frac{\partial\boldsymbol{r}(\ell,t)}{\partial s}\right)\left(1 - \left\|\frac{\partial\boldsymbol{r}(\ell,t)}{\partial s}\right\|^{-1}\right) \tag{A7}$$

To control the tethered satellite, the tension of the tether must be controlled at the reeling-out point. At this point, the elastic force corresponds to the tension. Hence, the boundary condition at $s=s_0$ is given by

$$\boldsymbol{r}(s_0,t) = \boldsymbol{0} \tag{A8}$$

$$EA\left(\frac{\partial\boldsymbol{r}(s_0,t)}{\partial s}\right)\left(1 - \left\|\frac{\partial\boldsymbol{r}(s_0,t)}{\partial s}\right\|^{-1}\right) = u(t)\frac{\partial\boldsymbol{r}(s_0,t)}{\partial s}\left\|\frac{\partial\boldsymbol{r}(s_0,t)}{\partial s}\right\|^{-1} \tag{A9}$$

Appendix B: Derivation of Control Region

When $\dot{z}(t)\geq 0$

The left-hand side of Eq. (19) is rewritten, where $\psi = \phi + \tan^{-1}\zeta_1/\Omega$,

$$\begin{aligned}(\text{left-hand side}) &= F\sqrt{\Omega^2+\zeta_1^2}\sin(\zeta_1 t+\psi) + [(h+k)/p]\\ &\quad\times(\Omega-\zeta_2)e^{\zeta_2 t} + [(h-k)/p](\Omega+\zeta_2)e^{-\zeta_2 t}\\ &\quad-\Omega z_{\min}\\ &\geq -F\sqrt{\Omega^2+\zeta_1^2} + [(h+k)/p](\Omega-\zeta_2)e^{\zeta_2 t}\\ &\quad+[(h-k)/p](\Omega+\zeta_2)e^{-\zeta_2 t} - \Omega z_{\min}\end{aligned} \tag{B1}$$

Here, we define $H(t)$ as

$$H(t) = -F\sqrt{\Omega^2+\zeta_1^2} + [(h+k)/p](\Omega-\zeta_2)e^{\zeta_2 t} + [(h-k)/p](\Omega+\zeta_2)e^{-\zeta_2 t} - \Omega z_{\min} \tag{B2}$$

To consider a safer side, we derive the condition of the initial state $\boldsymbol{X}_0$ that satisfies $H(t)\geq 0$ for all $t\geq 0$, and we utilize this condition as the switching condition.

When $h+k<0$, $H(t)\geq 0$ cannot be satisfied because the second term on the right-hand side of Eq. (B2) diverges to $-\infty$. When $h+k=0$, $H(t)$ is rewritten as

$$H(t) = -F\sqrt{\Omega^2+\zeta_1^2} + [(h-k)/p](\Omega-\zeta_2)e^{-\zeta_2 t} - \Omega z_{\min} \to -F\sqrt{\Omega^2+\zeta_1^2} - \Omega z_{\min} < 0 \qquad (t\to\infty) \tag{B3}$$

Therefore, $H(t)\geq 0$ cannot be satisfied in this case. Consequently, $h+k>0$ is a necessary condition of $H(t)\geq 0$ for all $t\geq 0$.

Next, we calculate $\dot{H}(t)$ and find t_1 that satisfies $\dot{H}(t_1)=0$.

$$\dot{H}(t) = \frac{h+k}{p}(\Omega-\zeta_2)\zeta_2 e^{\zeta_2 t} - \frac{h-k}{p}(\Omega+\zeta_2)\zeta_2 e^{-\zeta_2 t}$$

$$\dot{H}(0) = \frac{2\zeta_2(k\Omega - h\zeta_2)}{p} \tag{B4}$$

$$t_1 = \frac{1}{2\zeta_2}\ln\frac{(h-k)(\Omega+\zeta_2)}{(h+k)(\Omega-\zeta_2)} \tag{B5}$$

Because the time that satisfies $\dot{H}=0$ is unique and $H(t)\to+\infty$ $(t\to\infty)$, $H(t)\geq 0$ for all $t\geq 0$ can always be satisfied if the initial condition $\boldsymbol{X}_0$ satisfies either $H(t_1)\geq 0$ and $\dot{H}(0)<0$ or $H(0)\geq 0$ and $\dot{H}(0)\geq 0$.

If $\dot{H}(0)\geq 0$, that is, $k\Omega - h\zeta_2\geq 0$,

$$H(0) = -F\sqrt{\Omega^2+\zeta_1^2} + [(h+k)/p](\Omega-\zeta_2) + [(h-k)/p](\Omega+\zeta_2) - \Omega z_{\min}\geq 0 \tag{B6}$$

which can be rewritten as

$$2(h\Omega + k\zeta_2)/p - F\sqrt{\Omega^2+\zeta_1^2}\geq \Omega z_{\min} \tag{B7}$$

If $\dot{H}(0)<0$, that is, $k\Omega - h\zeta_2 < 0$,

$$H(t_1) = -F\sqrt{\Omega^2+\zeta_1^2} + [(h+k)/p](\Omega-\zeta_2)e^{\zeta_2 t_1} + [(h-k)/p](\Omega+\zeta_2)e^{-\zeta_2 t_1} - \Omega z_{\min}\geq 0 \tag{B8}$$

which can be rewritten as

$$\left[2\sqrt{(h^2-k^2)(\Omega^2-\zeta_2^2)}\Big/p\right]-F\sqrt{\Omega^2+\zeta_1^2}\geq\Omega z_{\min} \tag{B9}$$

When $\dot{z}(t)<0$

The derivation of this case is omitted because it is the same as in the case of $\dot{z}(t)\geq 0$. Then, $H(t)$ is defined as

$$H(t)=-F\sqrt{\Omega^2+\zeta_1^2}+[(h+k)/p](\Omega+\zeta_2)e^{\zeta_2 t}+[(h-k)/p](\Omega-\zeta_2)e^{-\zeta_2 t}-\Omega z_{\min} \tag{B10}$$

and the conditions of X_0 satisfying $H(t)\geq 0$ for all $t\geq 0$ are as follows:

$$h+k>0 \tag{B11}$$

If $\dot{H}(0)\geq 0$, that is, $k\Omega+h\zeta_2\geq 0$,

$$[2(h\Omega+k\zeta_2)/p]-F\sqrt{\Omega^2+\zeta_1^2}\geq\Omega z_{\min} \tag{B12}$$

If $\dot{H}(0)<0$, that is, $k\Omega+h\zeta_2<0$,

$$\left[2\sqrt{(h^2-k^2)(\Omega^2-\zeta_2^2)}\Big/p\right]-F\sqrt{\Omega^2+\zeta_1^2}\geq\Omega z_{\min} \tag{B13}$$

Consequently, the sufficient conditions of X_0 for Eqs. (19) and (20) being satisfied at any time can be summarized as Eqs. (21–23).

Appendix C: Discretization of the Tether

To solve the partial differential equation of the tether numerically, the equation is discretized. The length of the tether outside the mother ship is not constant because of deployment and retrieval. Hence, it is necessary to consider how the nodes representing the position of the tether should be chosen. In this paper, the whole tether is divided into segments of fixed length h, and the number of segments increases or decreases as the tethered satellite is deployed or retrieved. The position of the tethered satellite is denoted as the 0th node, and the position nearest to the mother ship is denoted as the nth node. The length from the nth node to the reeling-out point of the tether is denoted by $\Delta s(t)$. Using Eq. (A9), we can obtain the expression of $\Delta s(t)$ as follows:

$$\Delta s(t)=\{EA/[EA+u(t)]\}\|\boldsymbol{r}_n(t)\| \tag{C1}$$

The fluctuation of the number of nodes is determined using $\Delta s(t)$. That is, when $\Delta s(t)>1.5h$, one node is added, and when $\Delta s(t)<0.2h$, one node is deleted. Here, we must determine some quantities that describe the addition or deletion of one node. The quantities are the position and velocity of the node that newly appears or disappears, $\Delta s(t)$, and the number of nodes. These quantities are determined as follows.

1) When $\Delta s(t)>1.5h$ (Fig. C1) the distortions of the tether at the nth and $(n+1)$th nodes are considered to be the same. That is, we have the following equality.

$$\boldsymbol{r}_{n+1}/(\Delta s-h)=\boldsymbol{r}_n/\Delta s \tag{C2}$$

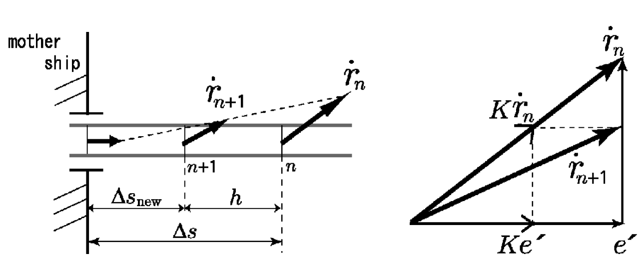

Fig. C1 Increase in the number of nodes.

Hence, the position vector of the $(n+1)$th node is determined as follows:

$$\boldsymbol{r}_{n+1}=[(\Delta s-h)/\Delta s]\boldsymbol{r}_n=K\boldsymbol{r}_n \tag{C3}$$

The tangential velocities of the tether at the nth and $(n+1)$th nodes are also considered to be the same. Then, the velocity vector at the $(n+1)$th node is determined by linear interpolation as follows, where $\boldsymbol{e}$ denotes the unit vector along $\boldsymbol{r}_n$ and $\boldsymbol{e}'=(\dot{\boldsymbol{r}}_n\cdot\boldsymbol{e})\boldsymbol{e}$.

$$\dot{\boldsymbol{r}}_{n+1}=K\dot{\boldsymbol{r}}_n+(1-K)\boldsymbol{e}' \tag{C4}$$

Then, one node is added, Δs is updated by $\Delta s_{\rm new}=\Delta s-h$ and n is increased by 1.

2) When $\Delta s(t)<0.2h$, one node is deleted and $\Delta s_{\rm new}=\Delta s+h$, and n is lessened by 1.

3) When neither $\Delta s(t)>1.5h$ nor $\Delta s(t)<0.2h$, the number of nodes is unchanged.

We solve the partial differential equation numerically by discretizing both s and t. Then, the result of discretization is expressed as

$$\boldsymbol{r}_i^{k+1}=F\left(\boldsymbol{r}_{i-1}^k,\boldsymbol{r}_i^k,\boldsymbol{r}_{i+1}^k\right)+G\left(\boldsymbol{r}_{i-1}^{k-1},\boldsymbol{r}_i^{k-1},\boldsymbol{r}_{i+1}^{k-1}\right) \tag{C5}$$

where $\boldsymbol{r}_i^k$ is the position of the ith node at time step k and F and G are appropriate functions, whose details are omitted. In the simulation, Δs is calculated first, and we check whether one node is added or deleted. As described by Eq. (C1), the control input can affect the motion of the tether through Δs, and $\boldsymbol{r}_i^{k+1}$ can be calculated using Eq. (C5).

These operations are repeated until the control input is updated. Because vibrational motion of the tether is much faster than librational motion of the tethered satellite, the time step for simulation using the partial differential equation is chosen to be smaller than the cycle for updating the control input. In this study, the time step was 0.001 s, and the duration of one cycle was 1 s.

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